Dark Energy from Brane-world Gravity

Roy Maartens

Institute of Cosmology & Gravitation, Portsmouth University,
Portsmouth PO1 2EG, UK

Summary. Recent observations provide strong evidence that the universe is accelerating. This confronts theory with a severe challenge. Explanations of the acceleration within the framework of general relativity are plagued by difficulties. General relativistic models require a "dark energy" field with effectively negative pressure. An alternative to dark energy is that gravity itself may behave differently from general relativity on the largest scales, in such a way as to produce acceleration. The alternative approach of modified gravity also faces severe difficulties, but does provide a new angle on the problem. This review considers an example of modified gravity, provided by brane-world models that self-accelerate at late times. [1]

1 Introduction

The current "standard model" of cosmology – the inflationary cold dark matter model with cosmological constant (LCDM), based on general relativity and particle physics (the minimal supersymmetric extension of the Standard Model) – provides an excellent fit to the wealth of high-precision observational data [1]. In particular, independent data sets from CMB anisotropies, galaxy surveys and SNe redshifts, provide a consistent set of model parameters. For the fundamental energy density parameters, this is shown in Fig. 1. The data indicates that the cosmic energy budget is given by

$$\Omega_\Lambda \approx 0.7\,, \quad \Omega_M \approx 0.3\,, \tag{1}$$

leading to the dramatic conclusion that the universe is undergoing a late-time acceleration. The data further indicates that the universe is (nearly) spatially flat, and that the primordial perturbations are (nearly) scale-invariant, adiabatic and Gaussian.

This standard model is remarkably successful, but we know that its theoretical foundation, general relativity, breaks down at high enough energies, usually taken to be at the Planck scale,

[1] Based on a talk at the 3rd Aegean Summer School, Chios, September 2005

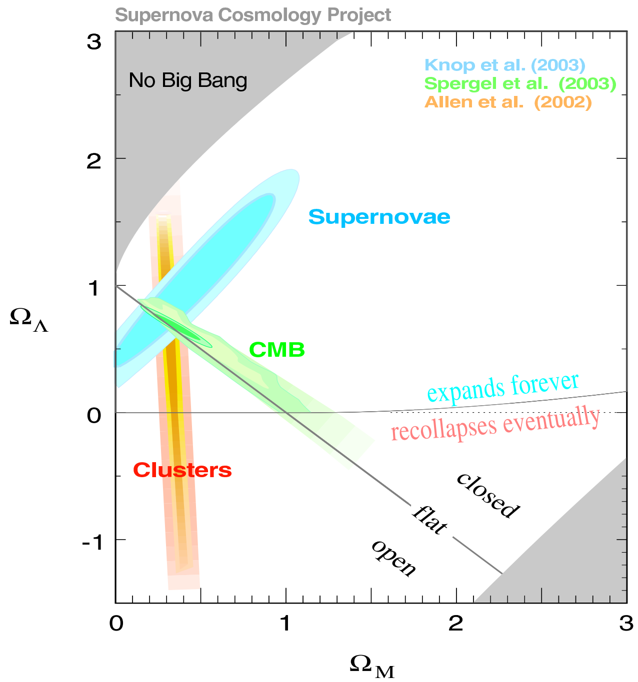

Fig. 1. Observational constraints in the $(\Omega_\Lambda, \Omega_M)$ plane (from [2]).

$$E \gtrsim M_p \sim 10^{16}\,\text{TeV}\,. \tag{2}$$

The LCDM model can only provide limited insight into the very early universe. Indeed, the crucial role played by inflation belies the fact that inflation remains an effective theory without yet a basis in fundamental theory. A quantum gravity theory will be able to probe higher energies and earlier times, and should provide a consistent basis for inflation, or an alternative that replaces inflation within the standard cosmological model.

An even bigger theoretical problem than inflation is that of the recent accelerated expansion of the universe. Within the framework of general relativity, the acceleration must originate from a dark energy field with effectively negative pressure ($w \equiv p/\rho < -\frac{1}{3}$), such as vacuum energy ($w = -1$) or a slow-rolling scalar field ("quintessence", $w > -1$). So far, none of the available models has a natural explanation.

For the simplest option of vacuum energy, i.e., the LCDM model, the incredibly small value of the cosmological constant

$$\rho_{\Lambda,\text{obs}} = \frac{\Lambda}{8\pi G} \sim H_0^2 M_P^2 \sim (10^{-33}\,\text{eV})^2 (10^{19}\,\text{GeV})^2 = 10^{-57}\,\text{GeV}^4, \tag{3}$$

$$\rho_{\Lambda,\text{theory}} \sim M_{\text{fundamental}}^4 > 1\ \text{TeV}^4 \gg \rho_{\Lambda,\text{obs}}\,, \tag{4}$$

cannot be explained by current particle physics. In addition, the value needs to be incredibly fine-tuned,

$$\Omega_\Lambda \sim \Omega_M \,, \tag{5}$$

which also has no natural explanation. Quintessence models attempt to address the fine-tuning problem, but do not succeed fully – and also cannot address the problem of how Λ is set exactly to 0. Quantum gravity will hopefully provide a solution to the problems of vacuum energy and fine-tuning.

Alternatively, it is possible that there is no dark energy, but instead a low-energy/ large-scale (i.e., "infrared") modification to general relativity that accounts for late-time acceleration. Schematically, we are modifying the geometric side of the field equations,

$$G_{\mu\nu} + G_{\mu\nu}^{\rm dark} = 8\pi G T_{\mu\nu} \,, \tag{6}$$

rather than the matter side,

$$G_{\mu\nu} = 8\pi G \left(T_{\mu\nu} + T_{\mu\nu}^{\rm dark}\right) \,, \tag{7}$$

as in general relativity.

It is important to stress that a consistent modification of general relativity requires a covariant formulation of the field equations in the general case, i.e., including inhomogeneities and anisotropies. It is not sufficient to propose ad hoc modifications of the Friedman equation, of the form

$$f(H^2) = \frac{8\pi G}{3}\rho \quad \text{or} \quad H^2 = \frac{8\pi G}{3} g(\rho) \,, \tag{8}$$

for some functions f or g. We can compute the SNe redshifts using this equation – but we *cannot* compute the density perturbations without knowing the covariant parent theory that leads to such a modified Friedman equation.

An infra-red modification to general relativity could emerge within the framework of quantum gravity, in addition to the ultraviolet modification that must arise at high energies in the very early universe. The leading candidate for a quantum gravity theory, string theory, is able to remove the infinities of quantum field theory and unify the fundamental interactions, including gravity. But there is a price – the theory is only consistent in 9 space dimensions. Branes are extended objects of higher dimension than strings, and play a fundamental role in the theory, especially D-branes, on which open strings can end. Roughly speaking, open strings, which describe the non-gravitational sector, are attached at their endpoints to branes, while the closed strings of the gravitational sector can move freely in the higher-dimensional "bulk" spacetime. Classically, this is realised via the localization of matter and radiation fields on the brane, with gravity propagating in the bulk (see Fig. 1).

The implementation of string theory in cosmology is extremely difficult, given the complexity of the theory. This motivates the development of phenomenology, as an intermediary between observations and fundamental theory. (Indeed, the development of inflationary cosmology has been a very valuable exercise in phenomenology.) Brane-world cosmological models inherit key

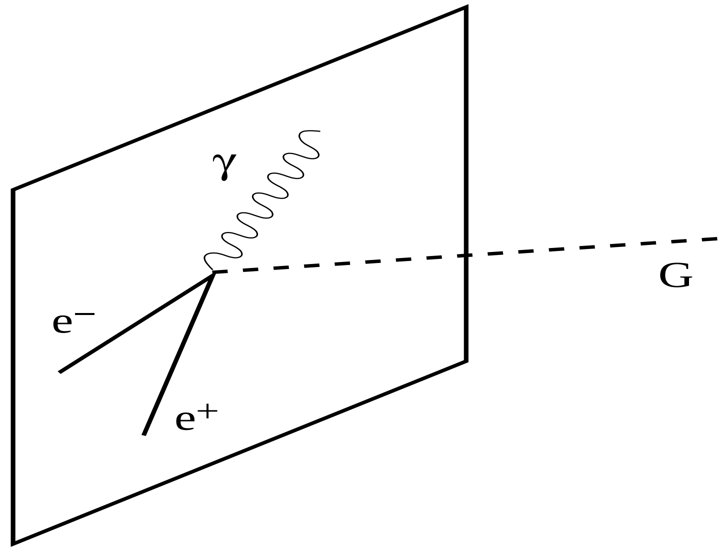

Fig. 2. The confinement of matter to the brane, while gravity propagates in the bulk (from [3]).

aspects of string theory, but do not attempt to impose the full machinery of the theory. Instead, drastic simplifications are introduced in order to be able to construct cosmological models that can be used to compute observational predictions (see [4] for reviews in this spirit). Cosmological data can then be used to constrain the brane-world models, and hopefully thus provide constraints on string theory, as well as pointers for the further development of string theory.

It turns out that even the simplest brane-world models are remarkably rich – and the computation of their cosmological perturbations is remarkably complicated, and still incomplete. Here I will describe brane-world cosmologies of Dvali-Gabadadze-Porrati (DGP) type [5]. These are 5-dimensional models, with an infinite extra dimension. (We effectively assume that 5 of the extra dimensions in the "parent" string theory may be ignored at low energies.)

2 KK modes of the graviton

The brane-world mechanism, whereby matter is confined to the brane while gravity accesses the bulk, means that extra dimensions can be much larger than in the conventional Kaluza-Klein (KK) mechanism, where matter and gravity both access all dimensions. The dilution of gravity via the bulk ef-

fectively weakens gravity on the brane, so that the true, higher-dimensional Planck scale can be significantly lower than the effective 4D Planck scale M_p.

The higher-dimensional graviton has massive 4D modes felt on the brane, known as KK modes, in addition to the massless mode of 4D gravity. From a geometric viewpoint, the KK modes can also be understood via the fact that the projection of the null graviton 5-momentum $p_a^{(5)}$ onto the brane is timelike. If the unit normal to the brane is n_a, then the induced metric on the brane is

$$g_{ab} = g_{ab}^{(5)} - n_a n_b \,,\; g_{ab}^{(5)} n^a n^b = 1 \,,\; g_{ab} n^b = 0 \,, \tag{9}$$

and the 5-momentum may be decomposed as

$$p_a^{(5)} = m n_a + p_a \,,\; p_a n^a = 0 \,,\; m = p_a^{(5)}\, n^a \,, \tag{10}$$

where $p^a = g^{ab} p_b^{(5)}$ is the projection along the brane, depending on the orientation of the 5-momentum relative to the brane. The effective 4-momentum of the 5D graviton is thus p_a. Expanding $g_{ab}^{(5)} p_{(5)}^a p_{(5)}^b = 0$, we find that

$$g_{\mu\nu} p^\mu p^\nu = -m^2 \,, \tag{11}$$

using coordinates $x^a = (x^\mu, y)$, where y is along the extra dimension. It follows that the 5D graviton has an effective mass m on the brane. The usual 4D graviton corresponds to the zero mode, $m = 0$, when $p_a^{(5)}$ is tangent to the brane.

The extra dimensions lead to new scalar and vector degrees of freedom on the brane. The spin-2 5D graviton is represented by a metric perturbation $h_{ab}^{(5)}$ that is transverse traceless:

$$g_{ab}^{(5)} \to g_{ab}^{(5)} + h_{ab}^{(5)} \,,\; h^{(5)a}{}_a = 0 = \nabla_b^{(5)}\, h^{(5)b}{}_a \,. \tag{12}$$

In a suitable gauge, $h_{ab}^{(5)}$ contains a 3D transverse traceless perturbation h_{ij} (where $x^\mu = (x^0, x^i)$), a 3D transverse vector perturbation Σ_i and a scalar perturbation β, which each satisfy the 5D wave equation:

$$h^i{}_i = 0 = \nabla_j h^{ij} \,,\; \nabla_i \Sigma^i = 0 \,, \tag{13}$$

$$(\nabla_\mu \nabla^\mu + \partial_y^2) \begin{pmatrix} \beta \\ \Sigma_i \\ h_{ij} \end{pmatrix} = 0 \,. \tag{14}$$

The 5 degrees of freedom (polarizations) in the 5D spin-2 graviton are felt on the brane as:

- a 4D spin-2 graviton h_{ij} (2 polarizations)
- a 4D spin-1 gravi-vector (gravi-photon) Σ_i (2 polarizations)
- a 4D spin-0 gravi-scalar β.

The massive modes of the 5D graviton are represented via massive modes in all 3 of these fields on the brane. The standard 4D graviton corresponds to the massless zero-mode of h_{ij}.

3 DGP type brane-worlds: self-accelerating cosmologies

Could the late-time acceleration of the universe be a gravitational effect?[2] An historical precedent is provided by attempts to explain the anomalous precession of Mercury's perihelion by a "dark planet". In the end, it was discovered that a modification to Newtonian gravity was needed.

An alternative to dark energy plus general relativity is provided by models where the acceleration is due to modifications of gravity on very large scales, $r \gtrsim H_0^{-1}$. It is very difficult to produce infrared corrections to general relativity by modifying the 4D Einstein-Hilbert action,

$$\int d^4x\,\sqrt{-g}\,R \;\to\; \int d^4x\,\sqrt{-g}\,f(R, R_{\mu\nu}R^{\mu\nu}, \dots)\,. \tag{15}$$

Typically, instabilities arise or the action has no natural motivation. The DGP brane-world offers a higher-dimensional approach to the problem, which effectively has infinite extra degrees of freedom from a 4D viewpoint.

Most brane-world models modify general relativity at high energies. The main examples are those of Randall-Sundrum (RS) type [6], where a Friedman-Robertson-Walker brane is embedded in an anti de Sitter bulk, with curvature radius ℓ. At low energies $H\ell \ll 1$, the zero-mode of the graviton dominates on the brane, and general relativity is recovered to a good approximation. At high energies, $H\ell \gg 1$, the massive modes of the graviton dominate over the zero mode, and gravity on the brane behaves increasingly in a 5D way. On the brane, the standard conservation equation holds,

$$\dot\rho + 3H(\rho + p) = 0\,, \tag{16}$$

but the Friedmann equation is modified by an ultraviolet correction:

$$H^2 = \frac{8\pi G}{3}\rho\left(1 + \frac{2\pi G\ell^2}{3}\rho\right) + \frac{\Lambda}{3}\,. \tag{17}$$

The ρ^2 term is the ultraviolet term. At low energies, this term is negligible, and we recover $H^2 \propto \rho + \Lambda/8\pi G$. At high energies, gravity "leaks" off the brane and $H^2 \propto \rho^2$. This 5D behaviour means that a given energy density produces a greater rate of expansion than it would in general relativity. As a consequence, inflation in the early universe is modified in interesting ways [4].

In the DGP case the bulk is 5D Minkowski spacetime. Unlike the AdS bulk of the RS model, the Minkowski bulk has infinite volume. Consequently, there is no normalizable zero-mode of the graviton in the DGP brane-world. Gravity leaks off the 4D brane into the bulk at large scales. At small scales, gravity is effectively bound to the brane and 4D dynamics is recovered to a

[2] Note that this would not remove the problem of explaining why the vacuum energy does not gravitate.

good approximation. The transition from 4- to 5D behaviour is governed by a crossover scale r_c; the weak-field gravitational potential behaves as

$$\Psi \sim \begin{cases} r^{-1} \text{ for } r \ll r_c \\ r^{-2} \text{ for } r \gg r_c \end{cases} \tag{18}$$

Gravity leakage at late times initiates acceleration – not due to any negative pressure field, but due to the weakening of gravity on the brane. 4D gravity is recovered at high energy via the lightest KK modes of the graviton, effectively via an ultralight metastable graviton.

The energy conservation equation remains the same as in general relativity, but the Friedman equation is modified:

$$\dot{\rho} + 3H(\rho + p) = 0\,, \tag{19}$$

$$H^2 - \frac{H}{r_c} = \frac{8\pi G}{3}\rho\,. \tag{20}$$

This shows that at early times, $Hr_c \gg 1$, the general relativistic Friedman equation is recovered. By contrast, at late times in a CDM universe, with $\rho \propto a^{-3} \to 0$, we have

$$H \to H_\infty = \frac{1}{r_c}\,. \tag{21}$$

Since $H_0 > H_\infty$, in order to achieve self-acceleration at late times, we require

$$r_c \gtrsim H_0^{-1}\,, \tag{22}$$

and this is confirmed by fitting SNe observations, as shown in Fig. 3. This comparison is aided by introducing a dimensionless cross-over parameter,

$$\Omega_{r_c} = \frac{1}{4(H_0 r_c)^2}\,, \tag{23}$$

and the LCDM relation,

$$\Omega_M + \Omega_\Lambda + \Omega_K = 1\,, \tag{24}$$

is modified to

$$\Omega_M + 2\sqrt{\Omega_{r_c}}\sqrt{1-\Omega_K} + \Omega_K = 1\,. \tag{25}$$

It should be emphasized that the DGP Friedman equation (20) is derived covariantly from a 5D gravitational action,

$$\int_{\rm bulk} d^5x\,\sqrt{-g^{(5)}}\,R^{(5)} + r_c \int_{\rm brane} d^4x\,\sqrt{-g}\,R\,. \tag{26}$$

LCDM and DGP can both account for the SNe observations, with the fine-tuned values $\Lambda \sim H_0^2$ and $r_c \sim H_0^{-1}$ respectively. This degeneracy may

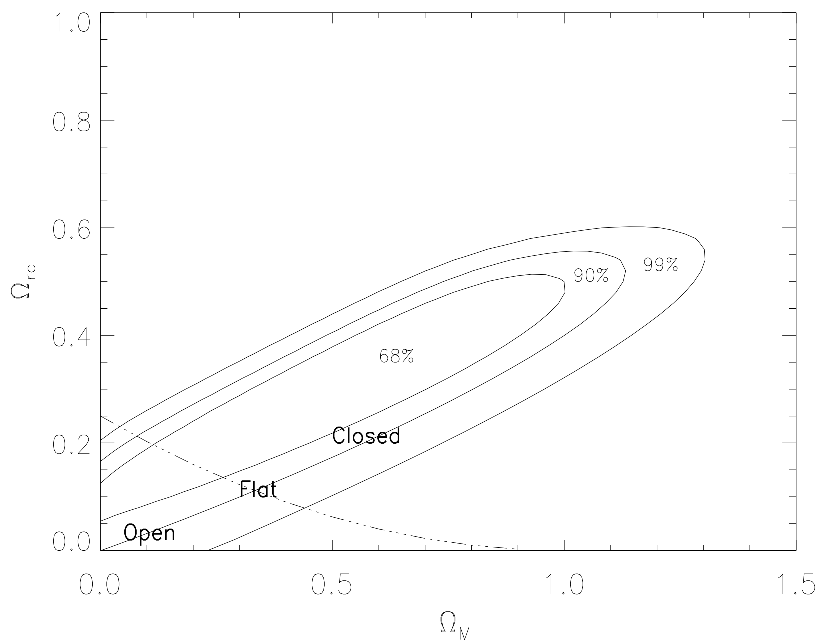

Fig. 3. Constraints from SNe redshifts on DGP models. (From [7].)

be broken by observations based on structure formation, since the two models suppress the growth of density perturbations in different ways [8, 9]. The distance-based SNe observations draw only upon the background 4D Friedman equation (20) in DGP models – and therefore there are quintessence models in general relativity that can produce precisely the same SNe redshifts as DGP [10]. By contrast, structure formation observations require the 5D perturbations in DGP, and one cannot find equivalent general relativity models [11].

For LCDM, the analysis of density perturbations is well understood. For DGP it is much more subtle and complicated. Although matter is confined to the 4D brane, gravity is fundamentally 5D, and the bulk gravitational field responds to and backreacts on density perturbations. The evolution of density perturbations requires an analysis based on the 5D nature of gravity. In particular, the 5D gravitational field produces an anisotropic stress on the 4D universe. Some previous results are based on inappropriately neglecting this stress and all 5D effects – as a consequence, the 4D Bianchi identity on the brane is violated, i.e., $\nabla^\nu G_{\mu\nu} \neq 0$, and the results are inconsistent.

When the 5D effects are incorporated [11], the 4D Bianchi identity is satisfied. (The results of [11] confirm and generalize those of [8].) The consistent modified evolution equation for density perturbations is

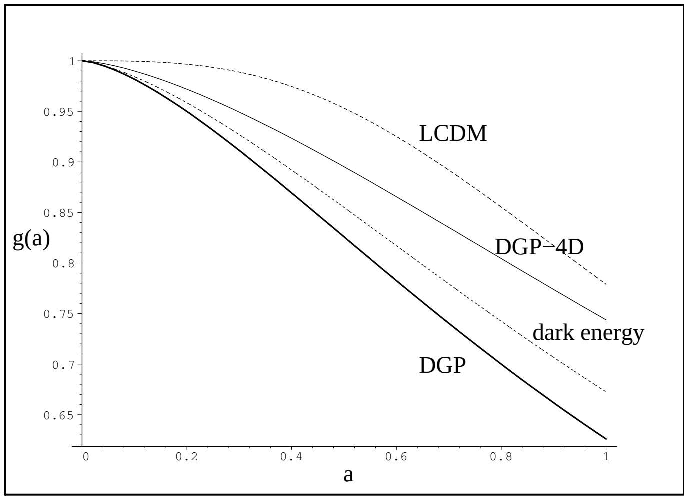

Fig. 4. The growth factor $g(a) = \Delta(a)/a$ for LCDM (long dashed) and DGP (solid, thick), as well as for a dark energy model with the same expansion history as DGP (solid, thick). DGP-4D (solid, thin) shows the incorrect result in which the 5D effects are set to zero. (From [11].)

$$\ddot{\Delta} + 2H\dot{\Delta} = 4\pi G \left\{ 1 - \frac{(2Hr_c - 1)}{3[2(Hr_c)^2 - 2Hr_c + 1]} \right\} \rho\Delta \,, \tag{27}$$

where the term in braces encodes the 5D correction. The linear growth factor, $g(a) = \Delta(a)/a$ (i.e., normalized to the flat CDM case, $\Delta \propto a$), is shown in Fig. 4.

It must be emphasized that these results apply on subhorizon scales. On superhorizon scales, where the 5D effects are strongest, the problem has yet to be solved. This solution is necessary before one can compute the large-angle CMB anisotropies – any prediction of the large-scale anisotropies without solving the 5D perturbation problem is unreliable.

It should also be remarked that the late-time asymptotic de Sitter solution in DGP cosmological models has a ghost problem [12], which makes the quantum vacuum unstable and which may have implications for the analysis of density perturbations. As a classical model, the DGP is covariant and consistent, and we effectively assume that the ghost problem will be solved by a quantum gravity ultraviolet completion of the model.

4 Conclusion

In conclusion, DGP brane-world models, which are inspired by ideas from string theory, provide a rich and interesting phenomenology for modified gravity. These models can account for the late-time acceleration without the need

for dark energy – gravity leakage from the 4D brane at large scales leads to self-acceleration. The 5D graviton, i.e., its KK modes, plays a crucial role, which has been emphasized in this article.

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References

1. See, e.g., D. Scott, arXiv:astro-ph/0510731.
2. R. A. Knop *et al.* [The Supernova Cosmology Project Collaboration], Astrophys. J. **598**, 102 (2003) [arXiv:astro-ph/0309368].
3. M. Cavaglia, Int. J. Mod. Phys. A **18**, 1843 (2003) [arXiv:hep-ph/0210296].
4. R. Maartens, Living Rev. Rel. **7**, 7 (2004) [arXiv:gr-qc/0312059];
   P. Brax, C. van de Bruck and A. C. Davis, Rept. Prog. Phys. **67**, 2183 (2004) [arXiv:hep-th/0404011];
   V. Sahni, arXiv:astro-ph/0502032;
   R. Durrer, AIP Conf. Proc. **782**, 202 (2005) [arXiv:hep-th/0507006];
   D. Langlois, arXiv:hep-th/0509231;
   A. Lue, Phys. Rept. **423**, 1 (2006) [arXiv:astro-ph/0510068];
   D. Wands, arXiv:gr-qc/0601078.
5. G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **484**, 112 (2000) [arXiv:hep-th/0002190];
   C. Deffayet, Phys. Lett. B **502**, 199 (2001) [arXiv:hep-th/0010186].
6. L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999) [arXiv:hep-th/9906064];
   P. Binetruy, C. Deffayet, U. Ellwanger and D. Langlois, Phys. Lett. B **477**, 285 (2000) [arXiv:hep-th/9910219].
7. C. Deffayet, S. J. Landau, J. Raux, M. Zaldarriaga and P. Astier, Phys. Rev. D **66**, 024019 (2002) [arXiv:astro-ph/0201164].
8. A. Lue, R. Scoccimarro and G. D. Starkman, Phys. Rev. D **69**, 124015 (2004) [arXiv:astro-ph/0401515];
   A. Lue and G. Starkman, Phys. Rev. D **67**, 064002 (2003) [arXiv:astro-ph/0212083].
9. Y. S. Song, Phys. Rev. D **71**, 024026 (2005) [arXiv:astro-ph/0407489];
   L. Knox, Y. S. Song and J. A. Tyson, arXiv:astro-ph/0503644;
   M. Ishak, A. Upadhye and D. N. Spergel, arXiv:astro-ph/0507184;
   I. Sawicki and S. M. Carroll, arXiv:astro-ph/0510364.
10. E. V. Linder, Phys. Rev. D **72**, 043529 (2005) [arXiv:astro-ph/0507263].
11. K. Koyama and R. Maartens, JCAP **0610**, 016 (2006) [arXiv:astro-ph/0511634].
12. D. Gorbunov, K. Koyama and S. Sibiryakov, Phys. Rev. D, to appear [arXiv:hep-th/0512097].